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# COMPUTING ev-DEGREE AND MULTIPLICATIVE ev-DEGREE INDICES OF CERTAIN CHEMICAL STRUCTURES

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## ABSTRACT
Recently, the *ev*-degree concept is defined in Graph Theory. In this study, we introduce the modified *ev*-degree Zagreb index, *ev*-degree inverse index, F-*ev*-degree index, reciprocal *ev*-degree product index, general *ev*-degree index of a graph. Also we propose some multiplicative *ev*-degree indices such as multiplicative *ev*-degree index, multiplicative modified *ev*-degree index, multiplicative F-*ev*-degree index, multiplicative *ev*-degree inverse index, multiplicative *ev*-degree product index, multiplicative reciprocal *ev*-degree product index, general multiplicative *ev*-degree index of a graph. We compute these *ev*-degree and multiplicative *ev*-degree indices for certain chemical structures.

**Keywords:** *ev*-degree indices, multiplicative *ev*-degree indices, networks, nanotubes.
***Mathematics Subject Classification:*** 05*C*05, 05*C*07, 05*C*90..

## 1. INTRODUCTION
Graph indices are important on the development of Theoretical Chemistry. Several graph indices were defined by using vertex degree concept, see [1]. Many graph indices have some applications in Chemistry, see [2,3].

Let $G$ be a finite, simple, connected graph with vertex set $V(G)$ and edge set $E(G)$. The degree $d_G(u)$ of a vertex $u$ is the number of vertices adjacent to $u$. The set of all vertices which adjacent to $v$ is called the open neighborhood of $v$ and denoted by $N(v)$. The closed neighborhood set of $v$ is the set $N[v] = N(v) \cup \{v\}$. Recently, Chellali et al. [4] defined the *ev*-degree concept in Graph Theory as follows:

**Definition:** Let $G$ be a connected graph and $e = uv \in E(G)$. The *ev*-degree of an edge $e$, denoted by $d_{ev}(e)$, equals the number of vertices of the union of the closed neighborhoods of $u$ and $v$. Clearly $d_{ev}(e)= d_G(u)+ d_G(v) - n_e$, where $n_e$ means the number of triangles in which edge $e$ lies in.

The *ev*-degree Zagreb index was introduced by Ediz in [5] and it is defined as

$$M_{ev}(G) = \sum_{e\in E(G)} d_{ev}(e)^2 .$$

We introduce the modified *ev*-degree Zagreb index of a graph $G$, defined as

$$^{m}M_{ev}(G) = \sum_{e\in E(G)} \frac{1}{d_{ev}(e)^2}$$

Also we define the ev-degree inverse index of a graph G as

$$I_{ev}(G) = \sum_{e\in E(G)} \frac{1}{d_{ev}(e)} .$$

We propose the *F-ev* degree index of a graph G, defined as

$$F_{ev}(G) = \sum_{e\in E(G)} d_{ev}(e)^3 .$$

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In [6], Ediz introduced the *ev*-degree Zagreb Randić or product index and it is defined as

$$P_{ev}(G) = \sum_{e \in E(G)} d_{ev}(e)^{-\frac{1}{2}}.$$

We now introduce the reciprocal *ev*-degree Randić or product index, defined as

$$RP_{ev}(G) = \sum_{e \in E(G)} d_{ev}(e)^{-\frac{1}{2}}.$$

We continue this generalization and introduce the general *ev*-degree index of a graph *G*, defined as

$$M^{a}{}_{ev}(G) = \sum_{e \in E(G)} d_{ev}(e)^{a} \tag{1}$$

where *a* is a real number.

Recently some graph indices were studied in [7, 8, 9, 10, 11, 12].

We propose some multiplicative *ev*-degree indices as follows:

The multiplicative *ev*-degree index of a graph *G* is defined as

$$M_{ev}II(G) = \prod_{e \in E(G)} d_{ev}(e)^{2}.$$

We introduce the multiplicative modified *ev*-degree index of a graph, defined as

$${}^{m}M_{ev}II(G) = \prod_{e \in E(G)} \frac{1}{d_{ev}(e)^{2}}.$$

We propose the multiplicative *F*-*ev*-degree index of a graph *G*, defined as

$$F_{ev}II(G) = \prod_{e \in E(G)} d_{ev}(e)^{3}.$$

We put forward the multiplicative *ev*-degree inverse index of a graph, defined as

$$I_{ev}II(G) = \prod_{e \in E(G)} \frac{1}{d_{ev}(e)}.$$

We introduce the multiplicative *ev*-degree product index of a graph, defined as

$$P_{ev}II(G) = \prod_{e \in E(G)} \frac{1}{\sqrt{d_{ev}(e)}}.$$

We propose the multiplicative reciprocal *ev*-degree product index of a graph *G*, defined as

$$RP_{ev}II(G) = \prod_{e \in E(G)} \sqrt{d_{ev}(e)}.$$

We continue this generalization and define the general multiplicative *ev*-degree index of a graph *G*, defined as

$$M_{ev}^{a}II(G) = \prod_{e \in E(G)} d_{ev}(e)^{a}. \tag{2}$$

Recently some multiplicative graph indices were studied, for example, in [13, 14, 15, 16, 17, 18, 19, 20, 21]. In this paper, some newly defined *ev*-degree indices and multiplicative *ev*-degree indices of certain structures such as networks, nanotubes are computed.

## 2. RESULTS FOR DOMINATING OXIDE NETWORKS

Dominating oxide network is important toll in chemistry, information science and physics. Mathematical and physical properties of this network have been studied in network and graph theory. The family of dominating oxide networks is symbolized by *DOX*(*n*). The molecular structure of a dominating oxide network is shown in Figure 1.

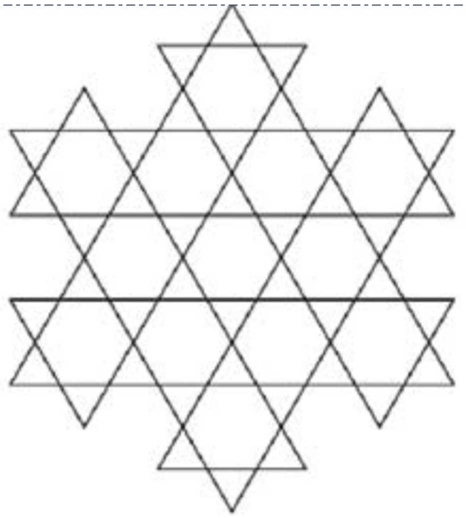

***Figure 1. The graph of DOX(2) network***

Let $G$=$DOX(n)$. From Figure 1, it is easy to see that the vertices of $G$ are either of degree 2 or 4. The graph $G$ has $54n^2 - 54n + 18$ edges. In $G$, by calculation, there are two types of edges based on the degrees of end vertices of each edge as follows:

$E_1 = \{uv \in E(G) \mid d_G(u) = 2, d_G(v) = 4\}$, $\quad |E_1| = 24n - 12$

$E_2 = \{uv \in E(G) \mid d_G(u) = d_G(v) = 4\}$, $\quad |E_2| = 54n^2 - 78n + 30$.

Clearly, we have $d_{ev}(e) = d_G(u) + d_G(v) - 1$. The partition of the edges with respect to their *ev*-degrees of end vertices for $G$ is given in Table 1.

***Table 1. The ve-degree position of DOX(n)***

| $d_{ev}(u) \setminus e \in E(G)$ | 5 | 7 |
|---|---|---|
| Number of edges | $24n - 12$ | $54n^2 - 78n + 30$ |

**Theorem 1.** The general ev-degree index of $DOX(n)$ is

$$M_{ev}^{a}(DOX(n)) = 54 \times 7^a n^2 + (24 \times 5^a - 78 \times 7^a)n + (30 \times 7^a - 12 \times 5^a) \qquad (3)$$

**Proof:** From equation (1) and Table 1, we deduce

$$M_{ev}^{a}(DOX(n)) = \sum_{e \in E(G)} d_{ev}(e)^a$$

$$= (24n - 12)5^a + (54n^2 - 78n + 30)7^a$$

$$= 54 \times 7^a n^2 + (24 \times 5^a - 78 \times 7^a)n + (30 \times 7^a - 12 \times 5^a)$$

We establish the following results by using Theorem 1.

**Corollary 1.1.** Let $DOX(n)$ be the family of dominating oxide network. Then

(i) $M_{ev}(DOX(n)) = 2646n^2 - 3222n + 1170.$

(ii) ${}^{m}M_{ev}(DOX(n)) = \frac{54}{49}n^2 - \frac{774}{1225}n + \frac{162}{1225}.$

(iii) $I_{ev}(DOX(n)) = \frac{54}{7}n^2 - \frac{222}{35}n + \frac{66}{35}.$

(iv) $F_{ev}(DOX(n)) = 18522n^2 - 23754n + 8790.$

(v) $P_{ev}\left(DOX(n)\right)=\frac{54}{\sqrt{7}}n^2+\left(\frac{24}{\sqrt{5}}-\frac{78}{\sqrt{7}}\right)n+\left(\frac{30}{\sqrt{7}}-\frac{12}{\sqrt{5}}\right).$

(vi) $RP_{ev}\left(DOX(n)\right)=54\sqrt{7}n^2+\left(24\sqrt{5}-78\sqrt{7}\right)n+\left(30\sqrt{7}-12\sqrt{5}\right).$

**Proof:** Put $a$ = 2, –2, –1 , 3, –½, ½ in equation (3), we obtain the desired results.

**Theorem 2.** The general multiplicative *ev*-degree index of *DOX*(*n*) is

$$M_{ev}^{a}II\left(DOX(n)\right)=5^{a(24n-12)}\times 7^{a(54n^2-78n+30)} \quad (4)$$

**Proof:** From equation (2) and Table 1, we derive

$$M_{ev}^{a}II\left(DOX(n)\right)=\prod_{e\in E(G)} d_{ev}(e)^a$$

$$=5^{a(24n-12)}\times 7^{a(54n^2-78n+30)}$$

From Theorem 2, we obtain the following results.

**Corollary 2.1.** Let *DOX*(*n*) be the family of dominating oxide network. Then

(i) $M_{ev}II\left(DOX(n)\right)=5^{2(24n-12)}\times 7^{2(54n^2-78n+30)}.$

(ii) ${}^{m}M_{ev}II\left(DOX(n)\right)=\left(\frac{1}{25}\right)^{24n-12}\times\left(\frac{1}{49}\right)^{54n^2-78n+30}.$

(iii) $F_{ev}II\left(DOX(n)\right)=5^{3(24n-12)}\times 7^{3(54n^2-78n+30)}.$

(iv) $I_{ev}II\left(DOX(n)\right)=\left(\frac{1}{5}\right)^{24n-12}\times\left(\frac{1}{7}\right)^{54n^2-78n+30}.$

(v) $P_{ev}II\left(DOX(n)\right)=\left(\frac{1}{\sqrt{5}}\right)^{24n-12}\times\left(\frac{1}{\sqrt{7}}\right)^{54n^2-78n+30}.$

(vi) $RP_{ev}II\left(DOX(n)\right)=\left(\sqrt{5}\right)^{24n-12}\times\left(\sqrt{7}\right)^{54n^2-78n+30}.$

**Proof:** Put $a$ = 2, –2, 3, –1, –½, ½ in equation (4), we obtain the desired results.

## 3. RESULTS FOR REGULAR TRIANGULATE OXIDE NETWORKS

The family of regular triangulate oxide networks is denoted by *TROX*(*n*), $n\geq 3$. The molecular structure of a regular triangulate oxide network is shown in Figure 2.

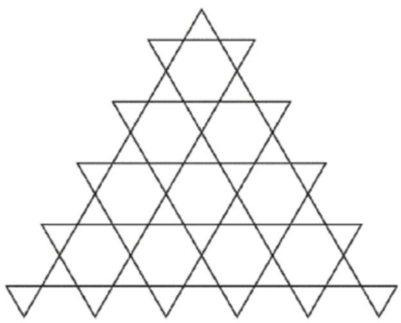

***Figure 2. The graph of TROX(5) network***

Let *H*=*TROX*(*n*). From Figure 2, we see that the vertices of *H* are either of degree 2 or 4. By calculation, in *H*, there three types of edges based on the degrees of end vertices of each edge as follows:

$E_1 = \{uv \in E(H) \mid d_H(u) = d_H(v) = 2\}$, $|E_1| = 2$.
$E_2 = \{uv \in E(H) \mid d_H(u) = 2, d_H(v) = 4\}$, $|E_2| = 6n$.
$E_3 = \{uv \in E(H) \mid d_H(u) = d_H(v) = 4\}$, $|E_3| = 3n^2 - 2$.

Clearly, we have $d_{ev}(e) = d_H(u) + d_H(v) - 1$. The partition of the edges with respect to their *ev*-degree of end vertices for *H* is given in Table 2.

| $d_{ev}(u) \setminus e \in E(H)$ | 3 | 5 | 7 |
|---|---|---|---|
| Number of edges | 2 | $6n$ | $3n^2 - 2$ |

Table 2. The *ev*-degree position of *TROX*(*n*)

**Theorem 3.** The general *ev*-degree index of *TROX*(*n*) is

$$M_{ev}^{a}(TROX(n)) = 3 \times 7^a n^2 + 6 \times 5^a n + (2 \times 3^a - 2 \times 7^a). \tag{5}$$

**Proof:** Let $H = TROX(n)$. From equation (1) and by using Table 2, we obtain

$$M_{ev}^{a}(TROX(n)) = \sum_{e \in E(H)} d_{ev}(e)^a$$
$$= 2 \times 3^a + 6n \times 5^a + (3n^2 - 2) \times 7^a$$
$$= 3 \times 7^a n^2 + 6 \times 5^a n + (2 \times 3^a - 2 \times 7^a).$$

By using Theorem 1, we establish the following results.

**Corollary 3.1.** Let *TROX*(*n*) be the family of regular triangulate oxide network. Then

(i) $M_{ev}(TROX(n)) = 147n^2 + 150n - 80.$

(ii) ${}^{m}M_{ev}(TROX(n)) = \frac{3}{49}n^2 + \frac{6}{25}n + \frac{80}{441}.$

(iii) $I_{ev}(TROX(n)) = \frac{3}{7}n^2 + \frac{6}{5}n + \frac{8}{21}.$

(iv) $F_{ev}(TROX(n)) = 1029n^2 + 750n - 632.$

(v) $P_{ev}(TROX(n)) = \frac{3}{\sqrt{7}}n^2 + \frac{6}{\sqrt{5}}n + \left(\frac{2}{\sqrt{3}} - \frac{2}{\sqrt{7}}\right).$

(vi) $RP_{ev}(TROX(n)) = 3\sqrt{7}n^2 + 6\sqrt{5}n + 2(\sqrt{3} - \sqrt{7}).$

**Proof:** Put $a = 2, -2, -1, 3, -½, ½$ in equation (5), we obtain the desired results.

**Theorem 4.** The general multiplicative *ev*-degree index of regular triangulate oxide network *TROX*(*n*) is

$$M_{ev}^{a}II(TROX(n)) = 3^{2a} \times 5^{6an} \times 7^{a(3n^2-2)} \tag{6}$$

**Proof:** Let $H = TROX(n)$. From equation (2) and Table 2, we derive

$$M_{ev}^{a}II(TROX(n)) = \prod_{e \in E(H)} d_{ev}(e)^a$$
$$= 3^{2a} \times 5^{6an} \times 7^{a(3n^2-2)}.$$

We obtain the following results by using Theorem 4.

**Corollary 4.1.** Let *TROX*(*n*) be the family of regular triangulate oxide network. Then

(i) $M_{ev}II(TROX(n)) = 3^4 \times 5^{12n} \times 7^{6n^2-4}.$

(ii) $${}^{m}M_{ev}II\left(TROX\left(n\right)\right)=\left(\frac{1}{3}\right)^{4}\times\left(\frac{1}{5}\right)^{12n}\times\left(\frac{1}{7}\right)^{6n^2-4}.$$

(iii) $$F_{ev}II\left(TROX\left(n\right)\right)=3^{6}\times 5^{18n}\times 7^{9n^2-6}.$$

(iv) $$I_{ev}II\left(TROX\left(n\right)\right)=\left(\frac{1}{3}\right)^{2}\times\left(\frac{1}{5}\right)^{6n}\times\left(\frac{1}{7}\right)^{3n^2-2}.$$

(v) $$P_{ev}II\left(TROX\left(n\right)\right)=\left(\frac{1}{3}\right)\times\left(\frac{1}{5}\right)^{3n}\times\left(\frac{1}{\sqrt{7}}\right)^{3n^2-2}.$$

(vi) $$RP_{ev}II\left(TROX\left(n\right)\right)=3\times 5^{3n}\times\left(\sqrt{7}\right)^{3n^2-2}.$$

**Proof:** Put $a$ = 2, –2, 3, –1, –½, ½ in equation (6), we get the desired results.

## 4. RESULTS FOR ARMCHAIR POLYHEX NANOTUBES

Carbon polyhex nanotubes are the nanotubes whose cylindrical surface is made up of entirely hexagons. These carbon nanotubes exist in nature with remarkable stability and possess very interesting thermal, electrical and mechanical properties. The family of armchair polyhex nanotubes is denoted by $TUAC_6[p, q]$, where $p$ is the number of hexagons in a row and $q$ is the number of hexagons in a column. A 2-dimensional network of $TUAC_6[p, q]$ is shown in Figure 3.

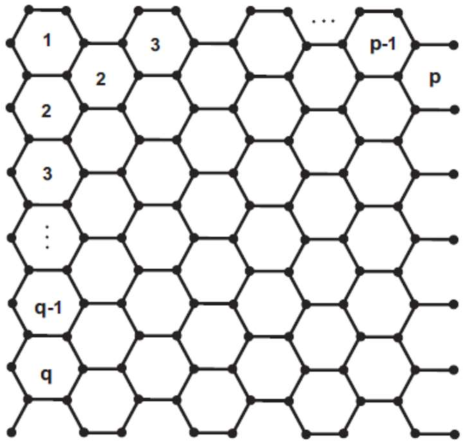


*Figure 3. The graph of TUAC₆[p, q] nanotube*

Let $A = TUAC_6[p, q]$. By calculation, $A$ has $2p(q+1)$ vertices and $3pq + 2p$ edges. In $A$, there are three types of edges based on degrees of end vertices of each edge. By calculation the edge degree partition of $A$ is as follows:

$E_1 = \{uv \in E(A) \mid d_A(u) = d_A(v) = 2\}$, $|E_1| = p$.
$E_2 = \{uv \in E(A) \mid d_A(u) = 2, d_A(v) = 3\}$, $|E_2| = 2p$.
$E_3 = \{uv \in E(A) \mid d_A(u) = d_A(v) = 3\}$, $|E_3| = 3pq - p$.

From Figure 3, we see that every edge of $TUAC_6[p, q]$ does not lie in a triangle. Thus $d_{ev}(e)= d_A(u)+ d_A(v)$. The partition of the edges with respect to their $ev$-degree of the end vertices for $A$ is given in Table 3.

*Table 3. The ev-degree partition of TUAC₆[p, q]*

| $d_{ev}(e) \setminus e \in E(A)$ | 4 | 5 | 6 |
|---|---|---|---|
| Number of edges | $p$ | $2p$ | $3pq - p$ |

**Theorem 5.** The general *ev*-degree index of $TUAC_6[p, q]$ is given by

$$M_{ev}^{a}(TUAC_6[p,q]) = 3\times 6^a pq + (4^a + 2\times 5^a - 6^a)p. \qquad (7)$$

**Proof:** From equation (1) and by using Table 3, we obtain

$$M_{ev}^{a}(TUAC_6[p,q]) = \sum_{e\in E(A)} d_{ev}(e)^a$$

$$= 4^a \times p + 5^a \times 2p + 6^a \times (3pq - p)$$

$$= 3\times 6^a pq + (4^a + 2\times 5^a - 6^a)p.$$

We obtain the following results by using Theorem 5.

**Corollary 5.1.** Let $TUAC_6[p, q]$ be the family of armchair polyhex nanotubes. Then

(i) $M_{ev}(TUAC_6[p,q]) = 108pq + 30p.$

(ii) $^{m}M_{ev}(TUAC_6[p,q]) = \frac{1}{12}pq + \frac{413}{3600}p.$

(iii) $I_{ev}(TUAC_6[p,q]) = \frac{1}{2}pq + \frac{29}{60}p.$

(iv) $F_{ev}(TUAC_6[p,q]) = 648pq + 98p.$

(v) $P_{ev}(TUAC_6[p,q]) = \frac{3}{\sqrt{6}}pq + \left(\frac{1}{2} + \frac{2}{\sqrt{5}} - \frac{1}{6}\right)p.$

(vi) $RP_{ev}(TUAC_6[p,q]) = 3\sqrt{6}pq + (2 + 2\sqrt{5} - \sqrt{6})p.$

**Proof:** Put $a$ = 2, –2, –1 , 3, –½, ½ in equation (7), we obtain the desired results.

**Theorem 6.** The general multiplicative *ev*-degree index of $TUAC_6[p, q]$ is

$$M_{ev}^{a}II(TUAC_6[p,q]) = 4^{ap}\times 5^{2ap}\times 6^{a(3pq-p)} \qquad (8)$$

**Proof:** Let $A = TUAC_6[p, q]$. By using equation (2) and Table 3, we derive

$$M_{ev}^{a}II(TUAC_6[p,q]) = \prod_{e\in E(A)} d_{ev}(e)^a$$

$$= 4^{ap}\times 5^{2ap}\times 6^{a(3pq-p)}.$$

We obtain the following results by using Theorem 6.

**Corollary 6.1.** Let $TUAC_6[p, q]$ be the family of armchair polyhex nanotubes. Then

(i) $M_{ev}II(TUAC_6[p,q]) = 4^{2p}\times 5^{4p}\times 6^{6pq-2p}.$

(ii) $^{m}M_{ev}II(TUAC_6[p,q]) = \left(\frac{1}{16}\right)^{p}\times\left(\frac{1}{25}\right)^{2p}\times\left(\frac{1}{36}\right)^{3pq-p}.$

(iii) $F_{ev}II(TUAC_6[p,q]) = 64^{p}\times 125^{2p}\times 216^{3pq-p}.$

(iv) $I_{ev}II(TUAC_6[p,q]) = \left(\frac{1}{4}\right)^{p}\times\left(\frac{1}{5}\right)^{2p}\times\left(\frac{1}{6}\right)^{3pq-p}.$

(v) $P_{ev}II(TUAC_6[p,q]) = \left(\frac{1}{2}\right)^{p}\times\left(\frac{1}{\sqrt{5}}\right)^{2p}\times\left(\frac{1}{\sqrt{6}}\right)^{3pq-p}.$

(vi) $RP_{ev}II\left(TUAC_6\left[p,q\right]\right) = 2^p \times 5^p \times \left(\sqrt{6}\right)^{3pq-p}$.

**Proof:** Put $a$ = 2, –2, 3, –1, –½, ½ in equation (8), we get the desired results.

## 5. RESULTS FOR ZIGZAG POLYHEX NANOTUBES

The family of zigzag polyhex nanotubes is symbolized by $TUZC_6[p, q]$, where $p$ is the number of hexagons in a row and $q$ is the number of hexagons in a column. A 2-dimensional networks of $TUZC_6\,[p, q]$ is presented in Figure 4.

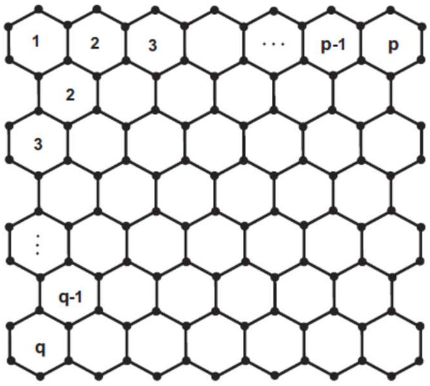


***Figure 4. The graph of $TUZC_6\,[p, q]$ nanotube***

Let $Z$ be the graph of zigzag polyhex nanotube $TUZC_6\,[p, q]$. By calculation, $Z$ has $2p(q+1)$ vertices and $3pq + 2p$ edges. In $Z$, there are two types of edges based on degrees of end vertices of each edge. By calculation, the edge degree partition of $Z$ is as follows:

$E_1 = \{uv \in E(Z) \mid d_Z(u) = 2, d_Z(v) = 3\}$, $|E_1| = 4p$.

$E_2 = \{uv \in E(Z) \mid d_Z(u) = d_Z(v) = 3\}$, $|E_2| = 3pq - 2p$.

From Figure 4, we observe that every edge of $TUZC_6[p, q]$ does not lie in a triangle. Thus $d_{ev}(e) = d_Z(u) + d_Z(v)$. The partition of the edges with respect to their $ev$-degree of the end vertices for $Z$ is given in Table 4.

***Table 4. The ev-degree partition of $TUZC_6[p, q]$***

| $d_{ev}(e) \setminus e \in E(Z)$ | 5 | 6 |
|---|---|---|
| Number of edges | $4p$ | $3pq - 2p$ |

**Theorem 7.** The general $ev$-degree index of $TUZC_6[p, q]$ is given by

$$M_{ev}^{a}\left(TUZC_6\left[p,q\right]\right) = 3 \times 6^a pq + \left(4 \times 5^a - 2 \times 6^a\right)p. \qquad (9)$$

**Proof:** Let $TUZC_6[p, q]$. By using equation (1) and Table 4, we deduce

$$M_{ev}^{a}\left(TUZC_6\left[p,q\right]\right) = \sum_{e \in E(Z)} d_{ev}(e)^a$$

$$= 5^a \times 4p + 6^a\left(3pq - 2p\right)$$

$$= 3 \times 6^a pq + \left(4 \times 5^a - 2 \times 6^a\right)p.$$

We establish the following results by using Theorem 7.

**Corollary 7.1.** Let $TUZC_6[p, q]$ be the family of zigzag polyhex nanotubes. Then

(i) $M_{ev}\left(TUZC_6\left[p,q\right]\right) = 108pq + 28p$.

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(ii) $\quad {}^{m}M_{ev}\left(TUZC_6\left[p,q\right]\right)=\frac{1}{12}pq+\frac{47}{450}p.$

(iii) $\quad I_{ev}\left(TUZC_6\left[p,q\right]\right)=\frac{1}{2}pq+\frac{7}{15}p.$

(iv) $\quad F_{ev}\left(TUZC_6\left[p,q\right]\right)=648pq+68p.$

(v) $\quad P_{ev}\left(TUZC_6\left[p,q\right]\right)=\frac{3}{\sqrt{6}}pq+\left(\frac{4}{\sqrt{5}}-\frac{2}{\sqrt{6}}\right)p.$

(vi) $\quad RP_{ev}\left(TUZC_6\left[p,q\right]\right)=3\sqrt{6}pq+\left(4\sqrt{5}-2\sqrt{6}\right)p.$

**Proof:** Put $a$ = 2, –2, –1 , 3, –½, ½ in equation (9), we obtain the desired results.

**Theorem 8.** The general multiplicative *ev*-degree index of zigzag $TUZC_6[p, q]$ is

$$M_{ev}^{a}II\left(TUZC_6\left[p,q\right]\right)= 5^{4ap}\times 6^{a(3pq-2p)}. \tag{10}$$

**Proof:** Let $Z = TUZC_6[p, q]$. From equation (2) and by using Table 4, we derive

$$M_{ev}^{a}II\left(TUZC_6\left[p,q\right]\right)= \prod_{e\in E(Z)} d_{ev}(e)^{a}$$

$$= 5^{4ap}\times 6^{a(3pq-2p)}.$$

From Theorem 8, we obtain the following results.

**Corollary 8.1.** Let $TUZC_6[p, q]$ be the family of zigzag polyhex nanotubes. Then

(i) $\quad M_{ev}II\left(TUZC_6\left[p,q\right]\right)=5^{8p}\times 6^{2(3pq-2p)}.$

(ii) $\quad {}^{m}M_{ev}II\left(TUZC_6\left[p,q\right]\right)=\left(\frac{1}{5}\right)^{8p}\times\left(\frac{1}{6}\right)^{2(3pq-2p)}.$

(iii) $\quad F_{ev}II\left(TUZC_6\left[p,q\right]\right)=5^{12p}\times 6^{3(3pq-2p)}.$

(iv) $\quad I_{ev}II\left(TUZC_6\left[p,q\right]\right)=\left(\frac{1}{5}\right)^{4p}\times\left(\frac{1}{6}\right)^{3pq-2p}.$

(v) $\quad P_{ev}II\left(TUZC_6\left[p,q\right]\right)=\left(\frac{1}{5}\right)^{2p}\times\left(\frac{1}{\sqrt{6}}\right)^{3pq-2p}.$

(vi) $\quad RP_{ev}II\left(TUZC_6\left[p,q\right]\right)=2^{2p}\times\left(\sqrt{6}\right)^{3pq-2p}.$

**Proof:** Put $a$ = 2, –2, 3, –1, –½, ½ in equation (10), we obtain the desired results.

## 6. RESULTS FOR TITANIA NANOTUBES

Titania is studied in material science. The family of titania nanotubes is symbolized by $TiO_2[m, n]$, where $m$ is the number of octagons $C_8$ in a row and $n$ is the number of octagons $C_8$ in a column. The graph of $TiO_2[m, n]$ is shown in Figure 5.

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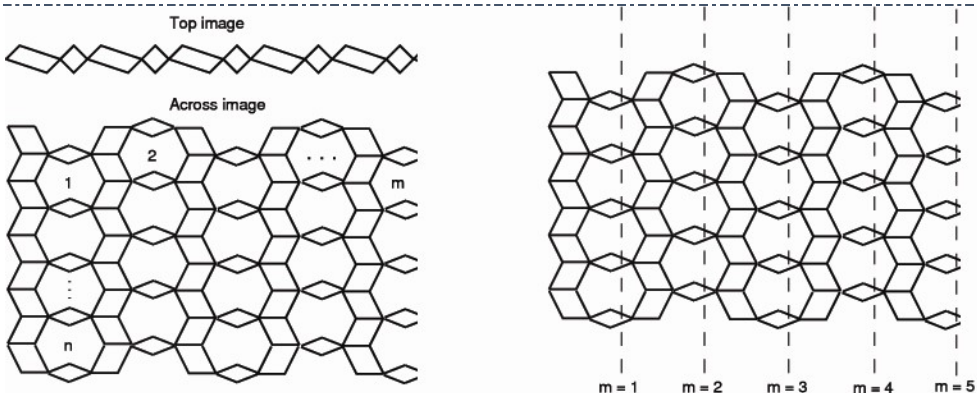


***Figure 5. The graph of $TiO_2[m, n]$ nanotube***

Let $T$ be the graph of titania nanotube $TiO_2[m, n]$. By calculation, $T$ has $6n(m+1)$ vertices and $10mn+8n$ edges. In $T$, by calculation, there are four types of edges based on the degree of end vertices of each edge as follows:

$E_1 = \{uv \in E(T) \mid d_T(u) = 2, d_T(v) = 4\}$, $\quad |E_1| = 6n$.

$E_2 = \{uv \in E(T) \mid d_T(u) = 2, d_T(v) = 5\}$, $\quad |E_2| = 4mn + 2n$.

$E_3 = \{uv \in E(T) \mid d_T(u) = 3, d_T(v) = 4\}$, $\quad |E_3| = 3n$.

$E_4 = \{uv \in E(T) \mid d_T(u) = 3, d_T(v) = 5\}$, $\quad |E_4| = 6mn - 2n$.

We observe that every edge of $TiO_2[m, n]$ does not lie in a triangle. Therefore $d_{ev}(e)= d_T(u)+ d_T(v)$. The partition of the edges with respect to their *ev*-degree of end vertices for $T$ is given in Table 5.

***Table 5. The ev-degree partition of $TiO_2[m, n]$***

| $d_{ev}(e) \setminus e \in E(T)$ | 6 | 7 | 7 | 8 |
|---|---|---|---|---|
| Number of edges | $6n$ | $4mn+2n$ | $2n$ | $6mn - 2n$ |

**Theorem 9.** The general ev-degree index of titania nanotubes $TiO_2$ is

$$M_{ev}^{a}(TiO_2) = \left(4\times 7^a + 6\times 8^a\right)mn + \left(6\times 6^a + 4\times 7^a - 2\times 8^a\right)n \qquad (11)$$

**Proof:** Let $T$=$TiO_2[m, n]$. From equation (1) and by Table 5, we deduce

$$M_{ev}^{a}(TiO_2) = \sum_{e\in E(T)} d_{ev}(e)^a$$

$$= 6^a \times 6n + 7^a \times (4mn + 2n) + 7^a \times 2n + 8^a (6mn - 2n)$$

$$= \left(4\times 7^a + 6\times 8^a\right)mn + \left(6\times 6^a + 4\times 7^a - 2\times 8^a\right)n$$

We establish the following results by using Theorem 9.

**Corollary 9.1.** Let $TiO_2[m, n]$ be the family of titania nanotubes. Then

(i) $M_{ev}(TiO_2) = 580mn + 284n.$

(ii) $^{m}M_{ev}(TiO_2) = \frac{275}{1568}mn + \frac{1021}{4704}n.$

(iii) $I_{ev}(TiO_2) = \frac{37}{28}mn + \frac{37}{28}n.$

(iv) $F_{ev}(TiO_2) = 4444mn + 1644n.$

(v) $P_{ev}(TiO_2) = \left(\frac{4}{\sqrt{7}} + \frac{3}{\sqrt{2}}\right)mn + \left(\frac{6}{\sqrt{6}} + \frac{4}{\sqrt{7}} - \frac{1}{\sqrt{2}}\right)n.$

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(vi) $RP_{ev}(TiO_2) = (4\sqrt{7} + 12\sqrt{2})mn + (6\sqrt{6} + 4\sqrt{7} - 4\sqrt{2})n.$

**Proof:** Put $a$ = 2, –2, –1 , 3, –½, ½ in equation (11), we get the desired results.

**Theorem 10.** The general multiplicative *ev*-degree index of $TiO_2[m, n]$ is

$$M_{ev}^{a}II(TiO_2) = 6^{6an} \times 7^{a(4mn+4n)} \times 8^{a(6mn-2n)} \quad (12)$$

**Proof:** Let $T = TiO_2[m, n]$. By using equation (2) and Table 5, we obtain

$$M_{ev}^{a}II(TiO_2) = \prod_{e \in E(T)} d_{ev}(e)^{a}$$

$$= 6^{6an} \times 7^{a(4mn+2n)} \times 7^{2an} \times 8^{a(6mn-2n)}$$

$$= 6^{6an} \times 7^{a(4mn+4n)} \times 8^{a(6mn-2n)}$$

We establish the following results from Theorem 10.

**Corollary 10.1.** Let $TiO_2[m, n]$ be the family of titania nanotubes. Then

(i) $M_{ev}II(TiO_2) = 6^{12n} \times 7^{8mn+8n} \times 8^{12mn-4n}.$

(ii) ${}^{m}M_{ev}II(TiO_2) = \left(\frac{1}{6}\right)^{12n} \times \left(\frac{1}{7}\right)^{8mn+8n} \times \left(\frac{1}{8}\right)^{12mn-4n}.$

(iii) $F_{ev}II(TiO_2) = 6^{18n} \times 7^{12mn+12n} \times 8^{18mn-6n}.$

(iv) $I_{ev}II(TiO_2) = \left(\frac{1}{6}\right)^{6n} \times \left(\frac{1}{7}\right)^{4mn+4n} \times \left(\frac{1}{8}\right)^{6mn-2n}.$

(v) $I_{ev}II(TiO_2) = \left(\frac{1}{6}\right)^{3n} \times \left(\frac{1}{7}\right)^{2mn+2n} \times \left(\frac{1}{8}\right)^{3mn-n}.$

(vi) $RP_{ev}II(TiO_2) = 6^{3n} \times 7^{2mn+2n} \times 8^{3mn-n}.$

**Proof:** Put $a$ = 2, –2, 3, –1, –½, ½ in equation (12), we obtain the desired results.

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